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THE TOPOLOGICAL STRUCTURE OF THE GLOBAL FOREIGN EXCHANGE MARKET DURING CRISES – COMPARATIVE NETWORK ANALYSIS[1]

Summary

Purpose – The aim of this article was to assess the changes in the topological structure of the currency market caused by two crises: the COVID-19 pandemic in 2020 and Russia's aggression against Ukraine in 2022. A network of major world currencies was analysed over three six-month sub-periods: the pandemic period 1.02–31.07.2020, the war period 1.02–31.07.2022 and the reference period 1.02–31.07.2021.

Research method – We have used the dynamic time warping (DTW) method for comparing time series. DTW distances between pairs of individual currencies were calculated, and, based on them, minimum spanning trees (MST) were constructed, whose topological characteristics were analysed.

Results – It turned out that the topological structure of the foreign exchange market varies in the sub-periods studied, and the analysed crises affected the currency network. In addition, the networks generated by the MST depend on the choice of base currency used to measure the value of all other currencies.

Originality/value/implications/recommendations – The significance of the results obtained lies in providing a description of the topological structure of the market during

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the observed crises. The detected hierarchical structures can be useful in theoretical descriptions of currencies and in the search for economic factors affecting specific groups of countries.

Keywords: foreign exchange market, minimum spanning tree, DTW distance

JEL classification: C10, D85, F31, G01

1. Introduction

The foreign exchange (FX) market is a global market where exchange rates between currencies are determined. The value of a currency is extremely important because it reflects one country's economic status, and the exchange rate represents the economic equilibrium of two countries. Exchange rates are among the most widely analysed economic indicators, and explaining their development in times of economic crises is an essential and challenging task. The currency market is a system that interacts with all sorts of information worldwide and is highly complex in its structure. For this reason, it is necessary to study not only the behaviour of individual currencies, but also the characteristics of the currency market as a whole, particularly the currency network's topological structure.

Our work aims to assess the changes in the topological structure of the FX market due to the COVID-19 pandemic and after the Russian invasion of Ukraine.

In the financial markets, network topology analysis provides effective methods for describing structure properties and is a broadly used analytical tool. We have investigated the network topology between currencies using a minimum spanning tree (MST) constructed from the dynamic time warping (DTW) distance measure. Topological features will be analysed for the resulting trees.

This article is organized as follows. Section 2 reviews selected literature on currency networks. In Section 3, we describe the data and methodology used. Section 4 reports and comments on the empirical results we have obtained. The conclusions are in Section 5.

2. Literature review

To create a dependency network for currencies, we can use various approaches such as the MST approach, the PMFG (planar maximally filtered graph) method or the correlation threshold method. The application of MST in finance was introduced by Mantegna [1999], who presented a network correlation analysis of the US stock market. The structural properties of the FX market have also been studied using this method.

MSTs have been applied to currency correlations by, for example, McDonald et al. [2005]. Ortega and Matesanz [2006], based on data from 28 countries, classified the effects of currency crises and constructed a hierarchical tree of the countries using MST. Górski et al. [2006] computed MSTs and discovered clustering effects for strong currencies. In the paper by Naylor et al. [2007], the authors used MST to create a topological network for 44 currencies from 1995–2001. The technique produced a robust, scale-free network. Górski et al. [2008] examined exchange rate correlation for 60 currencies. They presented MST graphs and discussed inverse power-like scaling. The topology of the FX market was also investigated by Kwapień et al. [2009]. For the exchange rates of 46 currencies from 1998–2008, they constructed networks for different base currencies. Their structure was not stable, although clusters existed that persisted over time.

The structural evolution of the Asian currency network was studied by Feng and Wang [2010]. A weaker correlation was observed between Asian currencies and the USD. The work of Wang et al. [2013] confirmed that the USD and EUR were the dominant world currencies, while the Asian cluster and the Latin American cluster were not stable. Rešovský et al. [2013] were also inspired by the MST methodology. They proved the centrality of economies linked to China (Singapore, Hong Kong).

Looking for articles related to international financial crises and FX market, one can find papers concerning the 2008 crisis in which the currency network is constructed using the MST approach [e.g. Fenn, 2010; Jang et al., 2011; Keskin et al., 2011; Wang, Xie, 2016; Kazemilari, Mohamadi, 2018; Chakraborty et al., 2020; Miśkiewicz, 2021; Zhang et al., 2022]. Jang et al. [2011] investigated the network properties of the foreign exchange market from 1990 to 2008 using the method of hierarchical taxonomy introduced by Mizuno et al. [2006]. They applied the correlation coefficients among foreign exchange rates as the distance to construct the minimal-spanning tree. Zhang et al. [2022] examined the topology of correlation networks using MST and PMFG. The PMFG retains the filtering features of the MST approach and includes additional links, cycles and cliques. An interesting example of the use of this method is the work of Wang and Xie [2016]. The authors used it to construct tail dependence networks and examine their topological, cluster and community structure properties. Kazemilari and Mohamadi [2018] studied the topological network of the FX market between 2005 and 2011 in relation to the subprime crisis. The region-based network analysis revealed significant changes in the constructed MSTs caused by the crisis. Also, Chakraborty et al. [2020] studied links between currencies and currency clusters during periods of severe international crises. The problem of globalisation during financial crises was analysed by Miśkiewicz [2021]. He found that in crises, exchange rate correlations increase, causing more cliques and higher ranks of nodes in the network.

Our work also employs the MST approach to explain the currency market network. Using MST requires a measure of similarity for currency time series, such as correlation coefficient [Naylor et al., 2007], partial correlation coefficient [Basnarkov et al., 2019], PCA distance [Fenn, 2010], or DTW distance [Wang et al., 2012]. There are different ways to convert the correlation coefficients into distances, but we chose to use the DTW approach, which is more suitable for comparing series with similar structures but shifted in time. The DTW method is popular in many fields, such as biometrics, bioinformatics, data mining, computer animation, and finance. It has also been used during the spread of the COVID-19 pandemic to study the topology of the network of similarities between currencies [Gupta, Chatterjee, 2020]. Wang et al. [2012] studied a network of 35 currencies before, during and after the subprime crisis in the US. They analysed changes in the structure of the MST, examined clusters of currencies and found that the USD and EUR were dominant. But the dollar gradually lost its position, while the euro acted as a stable center throughout the crisis. Gupta and Chatterjee [2020] examined the topology of the FX market using a measure based on the lead-lag relationship. The results indicated that as the crisis progressed, currencies became more strongly interconnected and the US dollar became more central.

3. Data basis and method of the analysis

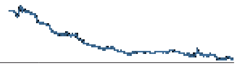
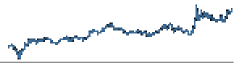
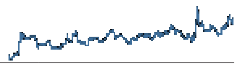
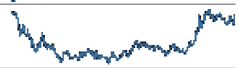
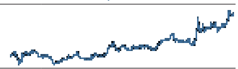
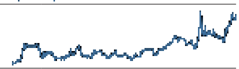
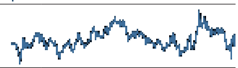
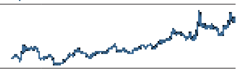
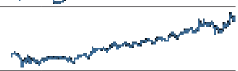
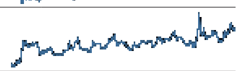
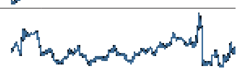
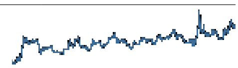
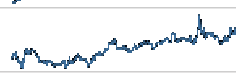
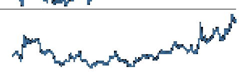
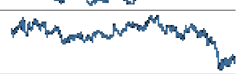
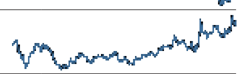
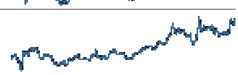
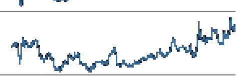
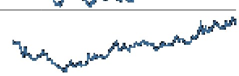
Our study is based on the daily FX time series of 36 currencies and four metals obtained from https://stooq.pl/ (see Table 1) over three periods:

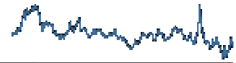
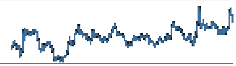
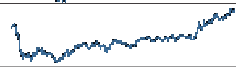
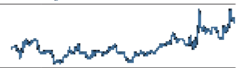
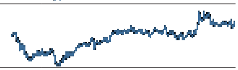
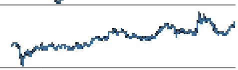
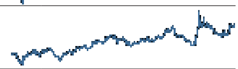
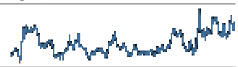
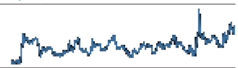
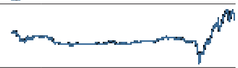
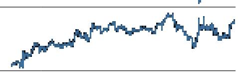
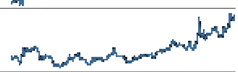
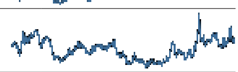
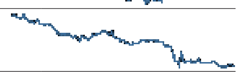
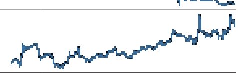
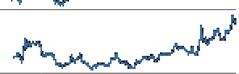
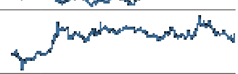
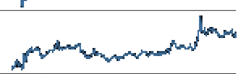
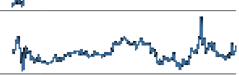
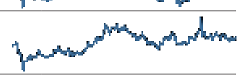
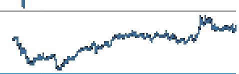
- 3.02–31.07.2020 (the COVID-19 pandemic outbreak period);
- 1.02–30.07.2021 (the reference period);
- 1.02–29.07.2022 (the period of the Russian invasion of Ukraine).

Prices on the FX market are quoted as exchange rates of the form X/Y, which indicate the amount of currency Y that can be received in exchange for one unit of currency X. Table 1 shows descriptive statistics and quotation trajectories for the analysed currencies and metals measured in PLN (Polish zloty). All currency time series were smoothed using a 5-day moving average.

TABLE 1

Descriptive statistics and course visualizations for the investigated currencies and metals quoted in PLN for the period 3.02.2020–29.07.2022

Code	Currency	Mean	S.d.	CV	Dynamics
ARS	Argentinian Peso	0.0452	0.0087	0.1923	
AUD	Australian Dollar	2.8635	0.1724	0.0602	
BGN	Bulgarian Lev	2.3259	0.0543	0.0234	
BRL	Brazilian Real	0.7574	0.0719	0.0949	
CAD	Canadian Dollar	3.0844	0.2050	0.0665	
CHF	Swiss Franc	4.2777	0.1896	0.0443	
CLP	Chilean Pesos	0.0050	0.0002	0.0378	
CNY	Chinese Yuan	0.6008	0.0412	0.0686	
CZK	Czech Koruna	0.1768	0.0084	0.0477	
DKK	Danish Krone	0.6110	0.0146	0.0238	
EGP	Egyptian Pound	0.2464	0.0107	0.0434	
EUR	Euro	4.5475	0.1055	0.0232	
GBP	UK Pound Sterling	5.2435	0.2356	0.0449	
HKD	Hong Kong Dollar	0.5116	0.0302	0.0590	
HUF	Hungarian Forint	0.0126	0.0003	0.0241	
IDR	Indonesian Rupiah	0.0003	0.0000	0.0564	
ILS	Israeli New Shekel	1.1993	0.0779	0.0650	
INR	Indian Rupee	0.0532	0.0023	0.0441	
ISK	Icelandic Krona	0.0304	0.0019	0.0630	

Code	Currency	Mean	S.d.	CV	Dynamics
JPY	Japanese Yen	0.0355	0.0014	0.0390	
KRW	South Korean Won	0.0034	0.0001	0.0299	
MXN	Mexican Peso	0.1916	0.0151	0.0788	
MYR	Malaysian Ringgit	0.9456	0.0411	0.0434	
NAD	Namibian Dollar	0.2552	0.0191	0.0749	
NOK	Norwegian Krone	0.4395	0.0243	0.0553	
NZD	New Zealand Dollar	2.6788	0.1394	0.0521	
PHP	Philippines Peso	0.0793	0.0025	0.0317	
RON	Romanian New Leo	0.9284	0.0159	0.0171	
RUB	Russian Rubles	0.0546	0.0079	0.1445	
SEK	Swedish Krona	0.4396	0.0139	0.0315	
SGD	Singaporean Dollar	2.9180	0.1538	0.0527	
THB	Thai Baht	0.1234	0.0040	0.0325	
TRY	Turkish Lira	0.4473	0.1143	0.2555	
TWD	Taiwan Dollar	0.1387	0.0076	0.0551	
USD	U.S. Dollar	3.9802	0.2487	0.0625	
XAG	Silver (ozt)	91.0279	12.8276	0.1409	
XAU	Gold (ozt)	7198.2815	552.3666	0.0767	
XPD	Palladium (ozt)	9008.7492	1113.2718	0.1236	
XPT	Platinum (ozt)	3912.2254	466.0661	0.1191	
ZAR	South Africa Rand	0.2551	0.0191	0.0750	

Source: authors' own elaboration based on data from https://stooq.pl.

Along with the rates of 36 currencies, the analysis also included the rates of four precious metals. Although the currency market and the precious metals market are separate markets, some researchers use metal prices as a base when constructing a correlation network for currencies [e.g. Górski et al., 2008]. In the remainder of this paper, we will present topological characteristics for networks built on the basis of XAU, among others. This will enable the comparability of the obtained results with those of other studies.

The choice of base currency (numéraire) is a significant problem in foreign exchange research. Currencies are priced against each other, so there is no independent numéraire. Any currency chosen as a base will be excluded from the network, but its internal patterns may indirectly affect overall patterns. It is important because different choices will produce different results if strong cross-correlations exist between currencies. This problem has no standard solution [Naylor et al., 2007; Wang et al., 2012; Gupta, Chatterjee, 2020]. In our analysis, the USD, EUR, XAU, and PLN act as numéraire currencies.

In order to quantify synchronisation between currencies, distances between all the possible pairs of exchange rates are calculated using the DTW method. They are computed from daily logarithm rate changes. We focus on a daily rate change R_t defined as:

$$R_t = \log\left(\frac{P_{t+1}}{P_t}\right) = \log P_{t+1} - \log P_t\,, \tag{1}$$

where P_t is the exchange rate at the time t.

DTW (Dynamic Time Warping) is a non-classical measure of distance. DTW is an algorithm for comparing time series, and it finds the smallest distance between two time series while allowing a time transformation for both [Bellman, Kalaba, 1959]. The DTW algorithm allows one to measure the similarity between two time series that may differ in time or speed. It is suitable for comparing the rather general shape of time series, even if shift or extension/narrowing graphs appear. DTW can be considered for time series of different lengths. In our work, we used time series of the same length.

The aim of the DTW method is to compare two time series, in our case $R_{1t} = (r_{11},\ldots,r_{1N})$ and $R_{2t} = (r_{21},\ldots,r_{2N})$.

First, a local cost matrix d of dimension $N \times N$ is defined. The d matrix consists of distance values $d(r_{1i}, r_{2j}) = \left|r_{1i} - r_{2j}\right|, i, j = 1,\ldots,N$. The elements of the d matrix correspond to shifts between the values of the considered time series R_1 and R_2.

Next, a warping path $w=(w_1,\dots,w_K), K\in\{N,\dots,2N-1\}$, is constructed. It consists of elements $w_k=(i_k, j_k)\in\{1,\dots, N\}\times\{1,\dots, N\}$. The warping path satisfies the boundary, monotonicity, and continuity conditions. That is:

1. boundary conditions: $w_1=(1\,,1)$, $w_K=(N\,,N)$;
2. monotonicity: $i_{k+1}-i_k\geq 0$ and $j_{k+1}-j_k\geq 0$;
3. continuity: $i_{k+1}-i_k\leq 1$ and $j_{k+1}-j_k\leq 1$.

To create the warping path, we start from $w_1 = (1, 1)$ and move forward at most one step up and/or right until we achieve the final element $w_K = (N\,,N)$. The total cost $D_w(R_1, R_2)$ of a warping path w between R_1 and R_2 is defined as

$$D_w(R_1,R_2)=\sum_{k=1}^{K}d(r_{1i_k},r_{2j_k}). \tag{2}$$

The optimal warping path has the minimum total cost related to the matching. Looking for the path that minimizes the warping cost, we obtain the DTW distance:

$$DTW(R_1,R_2)=\min_w\{D_w(R_1,R_2)\}. \tag{3}$$

In practice, the DTW method uses a dynamic programming algorithm with the following recursion: $\Gamma(i,j)=d(r_{1i},r_{2j})+\min\{\Gamma(i-1,j-1),\Gamma(i-1,j),\Gamma(i,j-1)\}$. Filling all the columns/rows of the matrix Γ, we finish at position (N,N), which is the DTW distance sought:

$$DTW(R_1,R_2)=\Gamma(N,N). \tag{4}$$

To discover the topological structure of the currency market, we use the MST concept proposed in the topological network analysis [Mantegna, 1999]. The currency network can be represented as a weighted graph $G = (V, E, w)$, which consists of nodes (vertices, V), edges between them (E), and weights (w). An undirected, connected and acyclic graph is called a tree. The tree that contains all the nodes of a graph $G = (V, E)$, and the set of edges of the tree is a subset of the set E, is a spanning tree of the graph G. In a weighted undirected graph, a spanning tree for which the sum of the weights of all edges is minimal is called a minimum spanning tree [Kruskal, 1956]. An MST compresses information about the network structure and simplifies the analysis by reducing the number of elements to be compared.

In our work, the MSTs are constructed using the DTW distance measure as weights. The Prim's node-based algorithm finds the MST. In this procedure, the tree starts at one arbitrary vertex and grows from that vertex with each step, checking the desired conditions [Prim, 1957]. Based on the topology of the network, a hierarchical structure of the community can be established by using the Girvan-Newman approach [Newman, Girvan, 2004]. This method recognises that, usually, edges between groups have a higher value of edge betweenness than edges within groups. Therefore, for all edges, the number of shortest paths passing through them is noted, and then the edges with the highest value of the betweenness measure are removed. The currencies are thus classified, and their groups are identified.

In the last step, the topological features of the obtained trees are analysed. The indices used for that purpose are:

- graph density – the percentage of edges present out of all possible edges in the network;
- average vertex degree – the average number of edges per node;
- mean distance – the average length of the shortest path among all pairs of vertices in the network (measures the efficiency of information flow);
- diameter – the length of the longest-shortest path connecting any pair of vertices (low values favor the transmission of information);
- maximum degree – the degree of the node with the largest number of edges adjacent to it;
- degree centrality – an indicator of graph centralization regarding the number of links of nodes (its high value indicates a high risk of nodes capturing everything that flows through the network);
- closeness centrality – an indicator of graph centralization regarding the closeness of nodes;
- assortativity – an indicator reflecting how nodes in a network are connected to each other (negative assortativity applies to star networks);
- modularity – an indicator that measures intra-community connections versus inter-community connections (modularity close to 1 means dense intra-community connections);
- closeness (at the vertex level) – the inverse of the distance between a vertex and all other vertices (the higher the closeness value, the more central a vertex is);
- strength (at the vertex level) – the sum of weights of the edges going from the vertex to its neighbours;
- betweenness (at the vertex level) – an indicator showing how often a vertex plays the role of a bridge on the shortest path connecting two other vertices (high betweenness indicates a node with more control over the network).

4. Results

In this section, we present the DTW and MSTs results using 36 major currencies and four metals, which we studied to investigate the topology of the networks among them in three time periods.

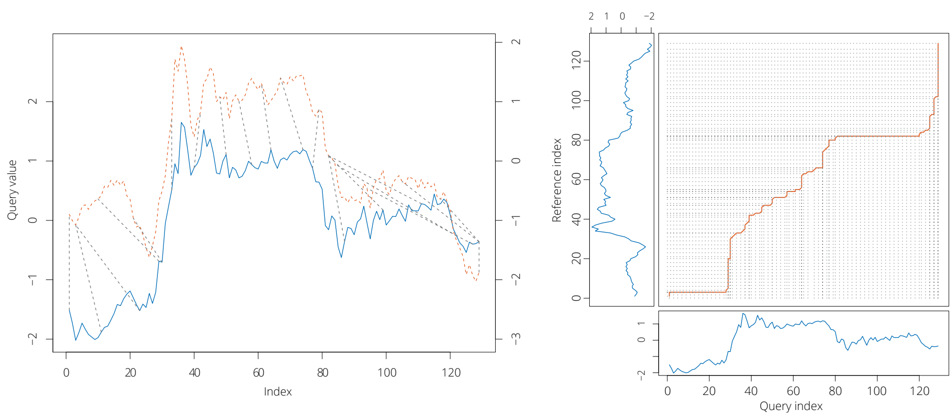
The MSTs were constructed for the USD, EUR, XAU, and PLN as a numéraire. Those currencies are chosen as bases because the USD and the EUR are the major currencies, and the XAU is the major metal. The PLN is not as important, but it allows us to get trees without having to remove other important currencies. We present the results obtained in a graphical form for the case of the PLN base and in tables for the other bases.

First, using the DTW algorithm, we calculated distance matrices to measure the similarity between each pair of currencies. Examples of time series alignment between USD/PLN and EUR/PLN in the period 3.02–31.07.2020 are shown in Chart 1: (a) for exchange rate quotes, (b) for currency returns. The lower right panel presents a three-way plot of the currency rates of return alignment with the optimal warping path in the middle. The shape of the warping path provides information about the pairwise correspondence of time points, and the distance value is the cost associated with the matching. Further analysis is based on the DTW distances between the rates of return for the currencies.

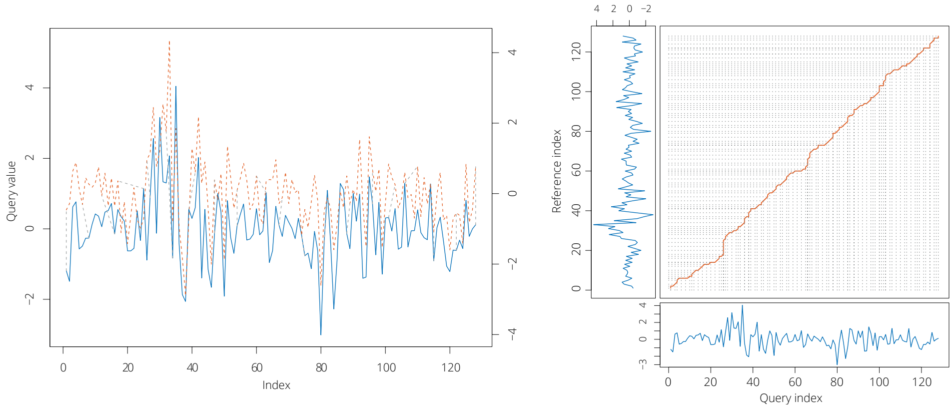
CHART 1

Time series alignment for USD/PLN and EUR/PLN in the period 3.02.–31.07.2020*

(a) for exchange rate quotes



(b) for currency returns



* Left panels: black lines for EUR, dashed grey lines for USD; right panels: query indexes for EUR, reference indexes for USD.

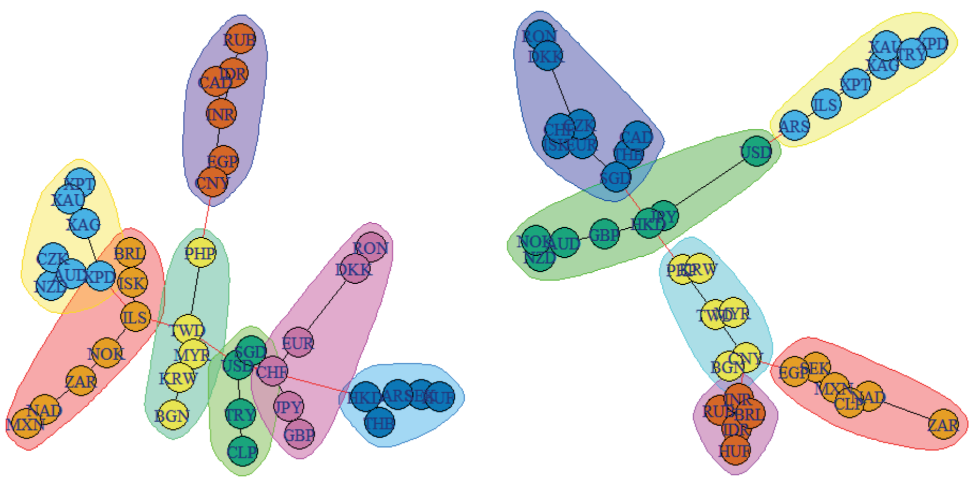
Source: authors' own elaboration in R.

The DTW method allowed the time series to be compared among themselves. The resulting distance matrices were then used to construct minimum spanning trees shown in Chart 2.

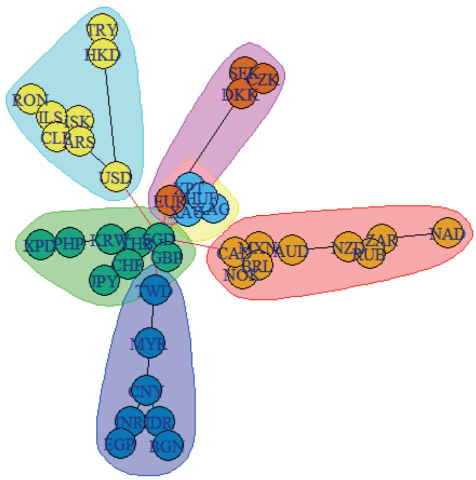
CHART 2

PLN-based minimum spanning trees for the currency networks

(a) Period 1: 3.02.–31.07.2020 (b) Period 2: 1.02.–30.07.2021



(c) Period 3: 1.02.–29.07.2022



Source: authors' own elaboration in R.

The constructed trees cover three time periods. The number of vertices in each constructed tree amounted to 40, the number of edges was 39, each graph density was 0.05, and the average vertex degree was 1.95. We performed community detection using the Girvan-Newman algorithm based on edge betweenness. The detected currency groups consist of more closely connected nodes with fewer connections between groups. In the panels of Chart 2, related currencies are displayed in the same color.

In the outbreak of the COVID-19 pandemic, seven subgroups of currencies were noted (see Chart 2(a)). Three of them occupied the central position in the constructed PLN-based MST: one with the USD at its center, the second with the CHF, and the other with TWD. The first cluster is composed of USD, SGD, CLP, and TRY. The second cluster is made of TWD, KRW, PHP, MYR (these are Asia-Pacific currencies), but also BGN. The third is the Europe cluster, and the neighbours of the CHF are European currencies, such as EUR, DKK, RON, GBP along with the JPY (the set of currencies highly correlated with each other). Also noteworthy is the cluster with metals XAU, XAG, XPT, and XPD, but also CZK, AUD, and NZD. The USD plays a significant role in currency intermediation, while the CHF group (along with the EUR) is not so centrally located. It confirms the thesis of the USD as the predominant currency in the FX market during crises (it is seen as a safe haven that can store value during a violent crisis). Previously, similar results were obtained, for example, by Wang et al. [2014] or Gupta and Chatterjee [2020].

From Chart 2(b), we can observe six clusters which are not similar to that seen in Chart 2(a). The following differences have been found: i) the central role plays the HKD cluster (with JPY, AUD, NZD, GBP, NOK, and USD; despite the latter currency being strongly attracted to the metal cluster), ii) the Asia-Pacific cluster has expanded to include China, but still plays an important role, iii) the EUR and other European currencies significantly distanced from the center.

The MST for the time of the Russian invasion of Ukraine (Chart 2(c)) has got a new center with the SGD playing the leading role (the central cluster consists of SGD, CHF, JPY, GBP, THB, KRW, PHP, and XPD). Closely related, albeit more peripherally located, are clusters centered around the USD, EUR and precious metals. A separate group of currencies of countries of great importance in the oil market deserves attention, e.g. CAD, BRL, NOK, MXN, RUB.

Building PLN-based MSTs is one of many possibilities. In the study, we also generated MSTs for other numéraire currencies. The resulting graphs are not shown here due to space limitations. For the networks obtained, the values of basic topological characteristics such as mean distance (average path length), diameter, maximum degree, degree centrality, closeness centrality, assortativity, and modularity were determined (see Table 2).

TABLE 2

Basic topological characteristics for networks in three time periods

Measure	base PLN			base USD			base EUR			base XAU		
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3
mean distance	2.95	3.65	2.27	3.62	3.31	3.63	4.11	4.29	3.81	2.51	3.04	2.72
diameter	6.15	8.72	5.04	8.37	7.86	8.90	9.91	9.69	8.60	6.58	8.26	6.21
maximum degree	5	5	9	6	6	4	6	6	5	6	4	5
degree centrality	0.08	0.08	0.18	0.10	0.10	0.05	0.10	0.10	0.08	0.10	0.05	0.08
closeness centrality	0.26	0.20	0.34	0.18	0.25	0.19	0.16	0.16	0.19	0.21	0.19	0.21
assortativity	0.03	-0.31	-0.17	-0.30	-0.35	-0.18	-0.41	-0.35	-0.13	-0.21	0.07	-0.21
modularity	0.70	0.70	0.68	0.70	0.70	0.69	0.70	0.70	0.70	0.69	0.70	0.69

Source: authors' own elaboration in R.

It should be noted that MSTs of the FX market present various topological properties in times of COVID-19 and war crises (period 1 and period 3) and beyond. For the PLN, EUR and XAU-based networks, the mean distances between nodes decreased, diameters showed smaller values, and the centralization of graphs increased during the outbreak of the pandemic and the outbreak of war. During both crises, the currencies were more linked to each other, and the structure of MSTs was more correlated. Bigger degree centrality values indicate that nodes are more likely to intercept everything that flows through the network. On the contrary, for the USD-based networks, the mean distances between nodes increased, the diameters showed bigger values, and the centralization of graphs decreased during the pandemic and the war.

Despite this, it should be noted that the currency network is characterised most often by negative assortativity (star structure with hubs). The relatively high modularity of the network partitioning reflects dense connections within communities and sparse connections between them.

Finally, local centrality indices were determined for selected currencies. Table 3 indicates the values of closeness, strength, and betweenness for each node.

TABLE 3

Local centrality indices for selected nodes in three time periods

Measure	base PLN			base USD			base EUR			base XAU		
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3
closPLN	–	–	–	0.007	0.010	0.010	0.005	0.005	0.005	0.013	0.011	0.011
closUSD	0.013	0.011	0.017	–	–	–	0.008	0.006	0.010	0.012	0.011	0.012
closEUR	0.009	0.008	0.015	0.006	0.014	0.011	–	–	–	0.017	0.014	0.016
closJPY	0.009	0.009	0.011	0.008	0.007	0.007	0.005	0.006	0.007	0.011	0.009	0.009
closGBP	0.007	0.009	0.014	0.010	0.009	0.008	0.007	0.007	0.005	0.013	0.013	0.012
closCHF	0.011	0.007	0.015	0.005	0.011	0.008	0.005	0.006	0.008	0.014	0.011	0.011
closRUB	0.006	0.006	0.009	0.006	0.009	0.007	0.007	0.007	0.007	0.010	0.010	0.010

Measure	base PLN			base USD			base EUR			base XAU		
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3
strPLN	–	–	–	1.225	1.287	1.940	0.707	0.795	1.463	0.829	1.192	1.023
strUSD	2.464	0.643	0.902	–	–	–	1.414	2.280	2.911	0.066	1.148	1.327
strEUR	0.592	2.910	0.578	0.700	1.933	1.506	–	–	–	2.406	1.529	0.563
strJPY	1.345	0.693	0.560	0.682	0.757	0.771	0.744	0.792	1.536	0.458	1.193	0.661
strGBP	0.700	1.471	0.547	2.691	0.717	0.701	1.400	2.371	0.776	1.008	1.844	0.538
strCHF	1.790	0.665	1.036	0.474	1.402	2.232	0.812	0.771	0.735	0.719	0.439	0.483
strRUB	0.721	0.794	0.718	2.144	0.774	2.000	1.393	1.555	1.299	1.381	0.689	0.688
betPLN	–	–	–	170	74	368	0	0	38	38	38	38
betUSD	415	224	234	–	–	–	332	178	480	0	143	75
betEUR	74	179	108	38	524	110	–	–	–	427	398	360
betJPY	38	0	0	0	0	0	0	0	38	0	38	0
betGBP	0	108	0	334	0	0	74	110	0	74	350	0
betCHF	176	0	38	0	224	143	0	0	0	38	0	0
betRUB	0	0	0	75	0	203	248	198	140	38	0	0

Source: authors' own elaboration in R.

It is worth noting that closeness values for the USD and EUR were higher than for other currencies in period 3, during the war. As for strength, it increased for USD/EUR, CHF/USD in period 3, and EUR in period 2 (especially for EUR/PLN). Higher betweenness was also recorded for USD (periods 1 and 3), EUR (period 2) and PLN/USD (period 3). The fact that some hubs appear during certain periods while others disappear (become marginal vertices) means that the network structure changes significantly over time. In our research, different currencies were allowed to play a dominant role unexpectedly.

5. Conclusions

In this study, we have presented the network topology between 40 major currencies and metals using MST and DTW concepts for the time of the COVID-19 pandemic outbreak (3.02–31.07.2020), the time of the Russian invasion of Ukraine (1.02–29.07.2022) and the reference period 1.02–30.07.2021. The clustered structure for the currency market and key currencies were obtained. The analysis was carried out for various reference (numéraire) currencies: PLN, USD, EUR and gold. Different bases generated different tree structures because the inclusion or exclusion of currencies from the sample gives different results. Similar results were obtained earlier by Mizuno et al. [2006] and Keskin et al. [2011].

A lot of attention has been paid in the literature to the fact that clusters obtained from MST are unstable [Marti et al., 2015]. The clustering instability results not only from the use of different base currencies but may also be partly due to the algorithm [Carlsson, Masmoli, 2010] or correlation measure used [Donnat et al., 2016]. For example, the Pearson linear correlation coefficient, which is often employed to measure distance between time series, is known for its sensitivity to outliers and is not appropriate for non-Gaussian distributions. Theoretical results ensuring the statistical reliability of hierarchical trees and networks are still not available [Marti et al. 2021] and further research work should continue in this area. Changing the method can cause a major change in grouping results. Consequently, this means huge variability in the formation of investors' portfolios. The findings obtained in the paper for practice are of limited relevance, also due to the short periods analysed. As a result, various structures were created, which, in the absence of a selection criterion, poses a significant problem. On the one hand, this means that we should not place particular emphasis on any particular MST score. The robustness of the method used should also be checked by comparison with other models.

On the other hand, the obtained results indicate that the two global crises under study had a major impact on exchange rates. However, it has been observed that the crises affected trees based on PLN, EUR and XAU currencies to a greater extent than trees based on USD. Our results confirm also that not only USD, EUR and CHF are important global currencies, but so are SGD, TWD and HKD. These currencies are often related to other currencies due to the geographic location and economic ties of countries.

The foreign exchange market, its volatility and structure, especially in the recent period of many perturbations of various etymologies, is a problem of great importance to central banks, government policymakers and investors as well. The results of our analysis demonstrate that dynamically changing economic relationships can be explained by topological network theory. The validity of the currently detected hierarchical structures can contribute to the theoretical descriptions of currencies and to the search for new economic factors affecting specific groups of countries.

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